Final Math106 (Take Home) Fall 2009

December 1-2009

Print Your Name:

All questions should be answered on this exam using the backs of the sheets if necessary. The exam has 6 pages, with 5 problems.

Show all your work. Good Luck !!

1	
2	
3	
4	
5	
Total	/100

Each problem is worth equally

NOTE: Any theorem you use has to be stated.

1. Solve using exponential matrices

$$X' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} X + \begin{pmatrix} 0 \\ e^t \end{pmatrix} \text{ with data } X(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

2. Solve the initial value problem using PIcard iteration

$$y' = 2t(1+y), \quad y(0) = 0.$$

(Get enough iterations to guess the general form, and if you can give the answer using induction)

3. a. Prove or disprove that a Hamiltonian system cannot have sinks or sources.
   b. Is the following system Hamiltonian

$$x' = (y^2 - y)(x + 1)$$
$$y' = x^2 - 1$$

   If it is not Hamiltonian, can you multiply the vector field by a function and make it into a Hamiltonian system.
   Find a conserved quantity of the new system. Will the solutions of the two systems be different, explain why or why not.

4. Analyze the following system:

$$x' = -y - x^3$$

$$y' = x - y^3$$

Does it have critical points besides of the origin?
Linearize around the origin. Is there a change in the behavior of the critical point at the origen between the linear system and the Non linear system
Draw the phase portraits and describe the stability at the origin.
Describe the behavior of the solution on the on the nullclines (draw them)
Is the system Hamiltonain. If not can you find a Lyapunov function. Is the origin asymptotically stable for the linear system? What about the nonlinear system?

5. Construct a suitable function of the form $ax^2 + cy^2$ where you need to determine a and c to figure out what type of critical point is the the origin for the following systems

$$\frac{dx}{dt} = -\frac{1}{2}x^3 + 2xy^2, \quad \frac{dy}{dt} = -y^3,$$

$$\frac{dx}{dt} = -x^3 + 2y^3, \quad \frac{dy}{dt} = -2xy^2,$$

$$\frac{dx}{dt} = x^3 - y^3, \quad \frac{dy}{dt} = 2xy^2 + 4x^2y + 2y^3.$$