

Final Math106B(Home) Winter 2008

February 19-2008

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| Print Your Name: |
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All questions should be answered on this exam using the backs of the sheets if necessary. The exam has 6 pages, with 5 problems.

Show all your work. Good Luck !!

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| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
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| Total | /100 |

Each problem is worth equally

NOTE: Any theorem you use has to be stated.

1. Solve

$$a. \quad e^{x^2} u_x + x u_y = 0$$

$$u(0, y) = y^2$$

Solve

$$b. \quad e^{x^2} u_x + x u_y + x u = 0$$

$$u(0, y) = y^2$$

2. Solve

$$u_{xx} + 10u_{xy} + 24u_{yy} = 0, \quad -\infty < x < \infty$$

$$u(x, 0) = x, \quad u_y(x, 0) = 0$$

3. Solve $u_{xx} + u_{yy} = 0$ in the exterior of the disk centered at the origin and radius $r = 2$

$$u(2, \theta) = \sin^2(\theta) + 1$$

4. Let $v_t - v_{xx} \geq \cos(x - \frac{\pi}{2})$, for $0 \leq x \leq \pi$, $0 < t < \infty$.
Suppose $v(0, t) \geq 0$, $v(\pi, t) \geq 0$, $v(x, 0) \geq \cos(x - \frac{\pi}{2})$. Show that

$$v(x, t) \geq (1 - e^{-t})\cos(x - \frac{\pi}{2})$$

5. a. Prove or give a counterexample : Can a series converge uniformly, but not pointwise.?

b. Given the series $\sum_{n=0}^{\infty} (-1)^n x^{2n}$.

i. Does the series converge pointwise in the interval $(-1, 1)$.

ii. Does the series converge uniformly in the interval $(-1, 1)$.

iii. Does the series converges in the L^2 sense in the interval $(-1, 1)$.

Prove or give a counterexample!

6. a. Solve

$$u_t - 2u_{xx} = 0, -\infty < x < \infty$$

$$u(x, 0) = 4e^{-x} + 1$$

b. Show by energy methods that the solution in a is unique.