Practice Problems 106B

February 11, 2008

1.**You need to know how to state the following definitions and theorems: NO PROOFS**

Concepts

[1.] State the maximum principle for Laplace's equation.
[2.] Under what conditions does a Fourier Series converge.
[3.] Definition for Uniform, pointwise and mean-square convergence..
[4.] You Should know how to use Max Principle to establish uniqueness.
[5.] You Should know how to use energy estimates to establish uniqueness

The problems of the exam will be of the following type. There will be about 7 problems

1. Solve

$$(1-x^2)u_x + u_y = 0$$

2. Solve

$$yu_x + xu_y = 0; \quad u(0,y) = e^{y^2}$$

3. Solve

$$u_x + u_y = 1$$

4. Classify second order equations.
5. Solve

$$u_{xx} - 3_{xy} - 4u_{tt} = 0, \quad u(x,0) = x^2, \ u_t(x,0) = e^x$$

1. Solve

$$u_t - u_{xx} = e^x, t > 0, \quad -\infty < x < \infty$$

$$u(x,0) = e^{-2x}, \quad -\infty < x < \infty$$

2 Solve

$$u_t - 4u_{xx} = 0, t > 0, \quad 0 < x < \infty$$

$$u(x,0) = e^{2x}, 0 < x < \infty.$$

$$u(0,t) = 2, t > 0$$

3. Use Fourier expansion to explain why a note produced by a violin rises when the string is tightened. What happens with the note when the length of the string is increased.

4.a. Solve

$$u_{tt} = u_{xx} - ru_t, \quad 0 < x < 2.$$

$$u(0,t) = u(2,t) = 0$$

$$u(x,0) = \sin 2\pi x, \quad u_t(x,0) = 0$$

Where r is a constant, $0 < r < \pi$.

5. Separate variables for the equation

$$tu_t = u_{xx} + 2u$$

with boundary conditions

$$u(0,t) = u(\pi,t) = 0$$

Show that there are an infinite number of solutions with initial condition $u(x,0) = 0$. That is no uniqueness!

6. Solve the diffusion equation $u_t = u_{xx}, \quad 0 < x < L$ with the mixed boundary conditions $u(0,t) = u_x(L,t) = 0$

7.Let $\phi(x) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{L}$ How do you get the A_n

8. Compute the Fourier sine , cosine and full Fourier series of the functions $F(x) = 1, F(x) = x$ on the interval $(0,1)$

9. Compute the Fourier sine series of $F(x) = x^2$ and , $F(x) = \sin(2\pi x)$ on the interval $(0,1)$.

10. Use the expression of the Fourier series of the function $F(x) = x$ to obtain a series giving the value of π