Final Practice Problems 106B

March 11, 2008

1. Review Practice problems for Midterm.

1.**You need to know how to state the following definitions and theorems: NO PROOFS**

Concepts

[1.] State the maximum principle for Laplace's equation.

[2.] Under what conditions does a Fourier Series converge.

[3.] Definition for Uniform, pointwise and mean-square convergence..

[4.] You Should know how to use Max Principle to establish uniqueness.

[5.] Mean value property for harmonic functions (page 162)

[6.] Be able to prof that a series converges in any of the senses in [3.] above.

1. Solve

$$u_t - u_{xx} = e^{-x}, t > 0, \;\; -\infty < x < \infty$$

$$u(x,0) = e^{-x}, \;\; -\infty < x < \infty$$

2.a. Solve

$$u_{tt} = u_{xx} - u_t, \;\; 0 < x < 2.$$

$$u(0,t) = u(2,t) = 0$$

$$u(x,0) = \sin 2\pi x, \;\; u_t(x,0) = 0$$

3. Show that if u, v satisfy Cauchy Riemann eqautions then u and v are harmonic.

4. Derive the Poisson formula to solve Laplace equation on a disk

5. Separate variables for the equation

$$tu_t = u_{xx} + 2u$$

with boundary conditions

$$u(0,t) = u(\pi,t) = 0$$

Show that there are an infinite number of solutions with initial condition $u(x,0) = 0$. That is no uniqueness!

6. Solve the diffusion equation $u_t = u_{xx}, \quad 0 < x < L$ with the mixed boundary conditions $u(0,t) = u_x(L,t) = 0$

7. Let $\phi(x) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{L}$ How do you get the A_n

8. Compute the Fourier sine , cosine and full Fourier series of the functions $F(x) = 1, F(x) = x$ on the interval $(0,1)$

9. Compute the Fourier sine series of $F(x) = x^2$ and ,$F(x) = \sin 2\pi x$ on the interval (0,1).

10. Use the expression of the Fourier series of the function $F(x) = x$ to obtain a series giving the value of π

10. Find the solutions that depend only on r of the equation

$$u_{xx} + u_{yy} + u_{zz} = 4u$$

(Hint use substitution $v = \frac{u}{r}$)

11. Solve

$$u_{xx} + u_{yy} = 1$$

in the annulus $a < r < b$ with u vanishing in both inner and outer boundaries.

12. Solve

$$u_{xx} + u_{yy} + u_{zz} = 1$$

in the spherical shell $a < r < b$ with $u = 0$ for $r = a$ and $\frac{\partial u}{\partial r} = 0$ on $r = b$. What happens when $a \to 0$.

13. Solve

$$u_{xx} + u_{yy} = 0$$

on the rectangle $0 < x < a, \quad 0 < y < b$ with the following boundary conditions

$$u_x = -a \text{ on } x = 0 \quad u_x = 0 \text{ on } x = a$$

$$u_y = -b \text{ on } y = 0 \quad u_y = 0 \text{ on } y = b$$

14. Suppose that u is a harmonic function in the disk $D = \{r < 2\}$ and that $u = 3\sin 2\theta + 1$ for $r = 2$ Without finding the solution answer the following questions.

a. Find the maximum value of u on $\bar{D}$.

b. Calculate $u(0)$

15.Problems in book: Page 154 :2, 4,6 Page 158: 1 Page 163 : 1,2,3.

16. Solve $u_{xx} + u_{yy} = 0,\ x^2 + y^2 > 4.$ with boundary condition $u(2, \theta) = 2$