Math 205 HW5:

1. Show that the hypothesis of σ- finitness cannot be omitted in The Radon-Nykodym Theorem.

2. Assume ν is a finite measure and μ is a *sigma*-finite measure on *sigma*-algebra $\mathcal{M}$, such that $\nu \ll \mu$. Let $g = \frac{d\nu}{d\mu} \in L^1(\mu)$ be the Radon-Nykodym derivative of ν respect to to μ. Then show that
If $f \in L^1(\nu)$, then $fg \in L^1(\mu)$, and $\int f d\nu = \int f g d\mu$.

3.a. Let ω be a *sigma*-finite measure and , μ, ν finite measures all on the same *sigma* algebra. Suppose that $\nu \ll \mu$ and $\mu \ll \omega$. Establish the chain rule:

$$\frac{d\nu}{d\omega} = \frac{d\nu}{d\mu}\frac{d\mu}{d\omega}(\omega a.e.)$$

b. We say that $\mu \equiv \nu$ if $\mu \ll \nu$ and $\nu \ll \mu$. Show that if $mu \equiv \nu$ then

$$\frac{d\nu}{d\mu}\frac{d\mu}{d\nu} = 1 \text{ a.e.}$$

Definitions that you will need for problems in Rudin

If X is a normed space we define the dual X^* as the space of all bounded linear functionals on X(transformations of the type $\Lambda : X \to \mathcal{C}$). If addition and scalar multiplication is defined in the obvious manner, then X^* is also a normed space with norm

$$\|\Lambda\| = \sup\{|\Lambda x| : x \in X, \|x\|_X \leq 1\}$$

Moreover you ca show it is a Banach space.