

**Math 205 HW5:**

1. a. Show that if  $0 < p < 1$  Then the reverse Hölder inequality holds.  
b. Use the reverse Hölder to show that the reverse Minkowski inequality holds.

2. Let  $\mu$  be a finite Borel measure on  $[0, \infty)$  such that

- $\mu \ll \lambda$ , and
- $\mu(B) = a\mu(aB)$  for each  $a \geq 1$  and for each Borel set  $B \in [0, \infty)$ , where  $aB = \{ab : b \in B\}$

If the Radon-Nikodym derivative  $\frac{d\mu}{d\lambda}$  is a continuous function, show that there exists a constant  $c \geq 0$  such that  $\frac{d\mu}{d\lambda}(x) = \frac{c}{x^2}$  for each  $x \geq 1$

3. Verify the following properties of signed measures

- a.  $\mu_1 \perp \mu_2$  then  $|\mu_1| \perp |\mu_2|$
- b.  $\nu \ll \mu$  and  $\mu \ll \omega$  then  $\nu \ll \omega$
- c.  $0 \leq \nu \leq \mu$  then  $\nu \ll \mu$
- d.  $\mu \ll 0$  then  $\mu = 0$