Math 207 Winter '07 Homework #7

6.3.2 (modified). Show that

$$\frac{1}{\Gamma(z+1)} = e^{\gamma z} \prod_{n=1}^{\infty} \left(\frac{n+z}{n}\right) e^{-z/n},$$

and deduce a formula for

$$\prod_{n=1}^{\infty} \frac{(n+a)(n+b)}{(n+c)(n+d)}$$

when $a+b=c+d$ and neither c nor d is a negative integer.

6.3.5. Use the fact that $\Gamma(s)\,\Gamma(1-s) = \pi/\sin\pi s$ to prove that

$$|\Gamma(1/2+it)| = \sqrt{\frac{\pi}{\cosh \pi t}}$$

when $t \in \mathbb{R}$.

6.3.6. Show that

$$1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} - \frac{1}{2}\log n \to \frac{\gamma}{2} + \log 2$$

as $n \to \infty$. [Hint: Add $1/2 + 1/4 + \cdots + 1/2n$.]

6.3.13. Prove that

$$\frac{d^2 \log \Gamma(s)}{ds^2} = \sum_{n=0}^{\infty} \frac{1}{(s+n)^2}$$

for all complex numbers $s \neq 0, -1, -2, \ldots$.

6.3 (not). Evaluate

$$\int_0^{\pi/2} (\cos\theta)^a\,(\sin\theta)^b\,d\theta$$

in terms of the gamma function.