PROJECTS

In what follows I will describe several possible projects. Choose one of them.// Here is a guideline for steps in project.

1. Decide on the model.
2. If possible give some analytical solution in simple cases.
3. Decide if there are bifurcations.
4. Find equilibria solutions and equilibrium points . Classify them.
5. Describe the trace-determinant plane.
4. Change parameters, see what are the changes in the solution.
5. You probably started with constant coefficients, now perhaps change some of the terms to given functions. (usually here you might add some external force or additional substance: check how things change depending on the type of the functions,ie if trigonometric or exponential, etc.
6. Graph different solutions, depending on parameters and changes you make. Compare the solutions and explain your results. Here you might be doing slope fields, direction fileds, phase lines, phase planes.
7. Anything you can thing that will make your discussion more interesting :)

Project 1: Drug kinetics.

The question here is to study the amount of drug introduced in a body tissue after a dose is introduced into the tissues by injection in the blood stream. We suppose the drug is injected continuously In the model suppose there is drug loss from the bloodstream, as the drug passes through the (kidneys)(renal clearance) and there is exchange of the drug between the bloodstream and the body tissues.

$$x = \text{ amount of drug in the blood}$$

$$y = \text{ amount of drug in the body tissue}$$

Assume that amounts of drug at time t are proportional to the drug present in each compartment (ie in bloodstream and in body tissue). Model should be of the form

$$x' = \{\text{inflow rate of drug into bloodstream}\} - \{\text{outflow rate of drug from bloodstream}\}$$

$$y' = \{\text{inflow rate of drug into tissues}\} - \{\text{outflow rate of drug from tissues}\}$$

where

$$\{\text{inflow rate of drug into bloodstream}\} = \{\text{inflow rate from tissue}\}$$

$$\{\text{outflow rate of drug from bloodstream}\} = \{\text{ renal clearance from bloodstream}\} +$$

$$\{\text{outflow rate from tissues}\},$$

Analyze the problem with different data. What happens if there is some external influence. What happens if the rates are not fixed, can you still get some answer numerically.

Ask yourself questions of the type:
In order for the drug to be effective we need a certain specific (you give the amount) of drug to enter in the tissue. What is the rates we need for that to happen before a fixed time.

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Project 2: Mass spring system.

In this project you have to analyze a spring-mass system. You should solve the equations if you can and then graph as much as possible

The most general for is

$$m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=F(t)$$

1. Write the equation as a system.
2. System without friction.
3. System with friction
4. Add a force. Analyze different possibilities. The main ones are, if you have no damping. In this case describe te phenomena of resonance and beats. In particular for resonance, try to explain why the Tacoma bridge fell. For beats try to explain how this happens.
5. How do the systems compare when there is no friction or when there is no damping. What happens if we let separately $b\to 0$ or $k\to 0$ or $m\to 0$. How do all of these solutions compare. Can you explain it.
6. Find examples where the solutions have oscillations other then the mass-spring system.

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Project 3: Population models

In class we discussed the logistic model for population

$$\frac{dP}{dt}=k(1-\frac{P}{N})P$$

1. Analyze this model in detail.
2. Give different data. Compare solutions, graphically and analytically.
3. Add different forces. Explain what the forces do.
4. Modify the model (be creative)
5. How would the population change if you compare it with a predator prey model, of the form

$$\frac{dP}{dt}=k(1-\frac{P}{N})P-F$$

$$\frac{dF}{dt} = -aF + bP$$

5. Vary all the parameters.

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Project 4: The Van der Pol equations
These equations are of the form

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -x + (1 - x^2)y$$

These equations are non linear, but the only nonlinearity is of the form $-x^2y$. It is easy to see that the origin is the only equilibrium point, since near zero the linear part of the system is dominant we can approximate the system by

$$\frac{dx}{dt} = y \tag{0.1}$$
$$\frac{dy}{dt} = -x + y$$

1. Try to explain (in a short fashion) why near zero the two solutions should similar.
2 Analyze system (0.1). Add parameters and different forces

$$\frac{dx}{dt} = ay + f(t) \tag{0.2}$$
$$\frac{dy}{dt} = -bx + cy + g(t)$$

3. Does the sign of the constants have any influence.
4. What about the size of the parameters.
5. What happens if you let one or more parameters tend to zero
6. How does this system compare with the mass-spring system.

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Proyect 5: The Lorentz system
For this proyect work out Problems from section2.5 Page 218 Problems 1,2,3,4,5.

1. Try to explain why linearization works.
2. Note that after linearization the variable z satisfies an equation that is independent of x, y If in the original equations on page 218, you linearize and vary parameters how do the phase planes of x, y vary.
3. What happens if any of the parameters is zero.